

85/100

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Agreed
second marker
JDP

PCEE 4000 PHASE TEST 2021

1. $V = \frac{V_0}{\sqrt{r^2 - 1}}$

$V\sqrt{r^2 - 1} = V_0$

$\frac{V_0}{V} = \sqrt{r^2 - 1}$ ✓

$\sqrt{\frac{V_0}{V}} = \sqrt{r^2 - 1}$ X

$\left(\sqrt{\frac{V_0}{V}}\right) = r^2$

$\sqrt{\left(\left(\sqrt{\frac{V_0}{V}}\right) + 1\right)} = r$ X

(2) ✓

2. a) $5(3x-2) - 2(4x-3) + 25 = 0$

$15x - 10 - 8x + 6 + 25 = 0$ ✓

$7x - 21 = 0$ X

$7x = 21$

$x = 3$ X

(1) ✓

b) $6x^2 + 13x - 5 = 0$ where $a=6$ $b=13$ $c=-5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

so $x = \frac{-13 \pm \sqrt{13^2 - 4 \times 6 \times (-5)}}{2 \times 6} = \frac{-13 \pm \sqrt{169}}{12} = \frac{-13 \pm 13}{12}$ X

and $x = \frac{-13 - \sqrt{49}}{12} = \frac{-13 - 7}{12} = \frac{-20}{12} = \frac{-5}{3}$ X

(1) ✓

c) $5 \times 10^{3x+2} = 215$

$10^{3x+2} = 43$ ✓

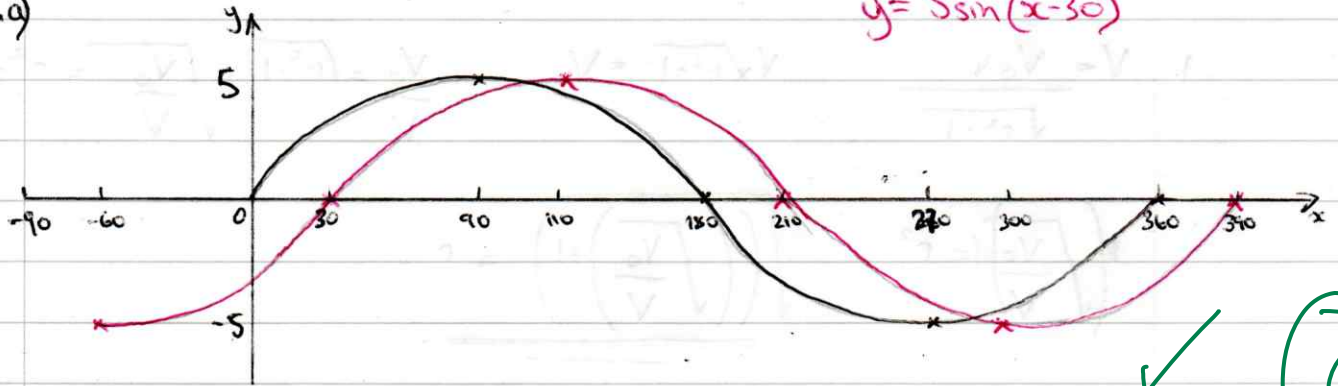
$3x+2 = \log_{10}(43)$ ✓

$3x = \log_{10}(43) - 2$ ✓

$x = \frac{\log_{10}(43) - 2}{3} = \underline{\underline{-0.12}}$ ✓

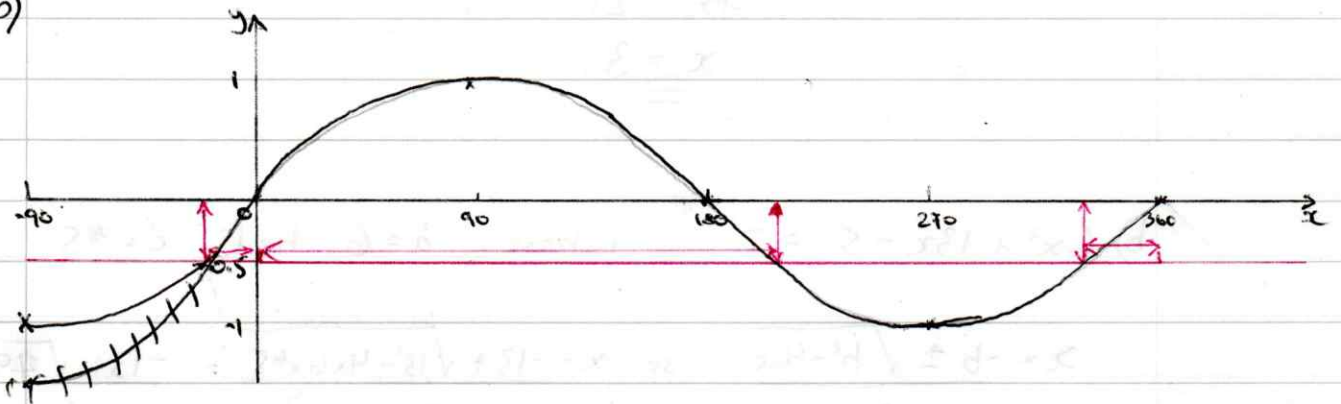
(6) ✓

3a)



Where $y = 5 \sin(x - 30)$ for $x = 0$, $x = 330$

b)



$$\sin 2x = -0.5 \quad \therefore 2x = \sin^{-1}(-0.5) = -30^\circ, (180+30), (360-30), (360+210), (360+330)$$

$$\text{so } x = 15^\circ, 105^\circ, 165^\circ, 285^\circ, 345^\circ$$

answers between 0° and 360°

4a) $Z_1 - Z_2 = (3 - j5) - (-2 + j4) = \underline{5 - 9j}$ ✓

b) $Z_1 Z_2 = (3 - j5)(-2 + j4) = \underline{14 + j22}$ ✓

c) $\frac{Z_1}{Z_2} = \frac{(3 - j5)(-2 - j4)}{(-2 + j4)(-2 - j4)} = \frac{-6 - j12 + j10 + j^2 20}{4 + j8 - j8 - j^2 16} = \frac{-6 - j2 + (-1)20}{4 - (-1)16} = \frac{-26 - j2}{20} = \underline{\underline{\frac{-26}{20} - j\frac{2}{20}}}$ ✓

$$5. a) 3A - 2B = 3 \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} - 2 \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ 6 & -9 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -4 & 12 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 10 & -21 \end{bmatrix} \checkmark$$

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$$b) AB = \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 4 \times 5 + (-1) \times (-2) & 4 \times 0 + (-1) \times 6 \\ 2 \times 5 + (-3) \times (-2) & 2 \times 0 + (-3) \times 6 \end{bmatrix} = \begin{bmatrix} 20 + 2 & 0 - 6 \\ 10 + 6 & 0 - 18 \end{bmatrix} = \begin{bmatrix} 22 & -6 \\ 16 & -18 \end{bmatrix} \checkmark$$

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$$c) BA = \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 \times 4 + 0 \times 2 & 5 \times (-1) + 0 \times (-3) \\ -2 \times 4 + 6 \times 2 & -2 \times (-1) + 6 \times (-3) \end{bmatrix} = \begin{bmatrix} 20 + 0 & -5 + 0 \\ -8 + 12 & 2 - 18 \end{bmatrix} = \begin{bmatrix} 20 & -5 \\ 4 & -16 \end{bmatrix} \therefore AB \neq BA \checkmark$$

$$d) \det(A) = (4 \times -3) - (-1 \times 2) = -10 \checkmark$$

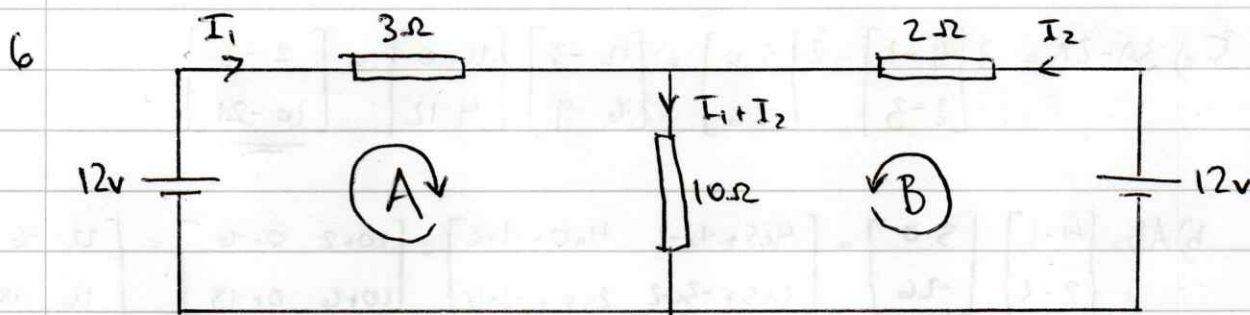
$$\text{Rearrangement step} = \begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix} \checkmark$$

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$$-\frac{1}{10} \begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ \frac{2}{10} & -\frac{4}{10} \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Simplify

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Where $V = IR$

In A

$$12 = 3 \times I_1 + (10 \times I_1 + 10 \times I_2)$$

$$12 = 13 I_1 + 10 I_2$$

In B

$$12 = 2 \times I_2 + (10 \times I_1 + 10 \times I_2)$$

$$12 = 10 I_1 + 12 I_2$$

$$\text{So } 12 = 13 I_1 + 10 I_2$$

$$12 = 10 I_1 + 12 I_2$$

$$\det = (13 \times 12) - (10 \times 10) = 56$$

$$\text{Augmented step} = \begin{bmatrix} 12 & -10 \\ -10 & 13 \end{bmatrix}$$

$$\text{So } \frac{1}{56} \begin{bmatrix} 12 & -10 \\ -10 & 13 \end{bmatrix} \begin{bmatrix} 13 & 10 \\ 10 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{56} \begin{bmatrix} 12 & -10 \\ -10 & 13 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

$$\frac{1}{56} \begin{bmatrix} 12 \times 13 + 10 \times 10 & 12 \times 10 + 10 \times 12 \\ -10 \times 13 + 13 \times 10 & -10 \times 10 + 13 \times 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{56} \begin{bmatrix} 12 \times 12 + 10 \times 12 \\ -10 \times 12 + 13 \times 12 \end{bmatrix}$$

$$\frac{1}{56} \begin{bmatrix} 156 + 100 & 120 + 120 \\ -130 + 130 & -100 + 156 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{56} \begin{bmatrix} 144 + 120 \\ -120 + 156 \end{bmatrix}$$

$$\frac{1}{56} \begin{bmatrix} 56 & 0 \\ 0 & 56 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{56} \begin{bmatrix} 264 \\ 36 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24/56 \\ 36/56 \end{bmatrix} = \begin{bmatrix} 12/28 \\ 18/28 \end{bmatrix} = \begin{bmatrix} 6/14 \\ 9/14 \end{bmatrix} \Rightarrow \begin{bmatrix} 3/7 \\ 9/14 \end{bmatrix}$$

so $I_1 @ 3\Omega = 3/7 A$ ✓

$I_2 @ 2\Omega = 9/14 A$ ✓

$I @ 10\Omega = I_1 + I_2 = 12/14 A = 6/7 A$ ✗

14 ✓

7a) so where $x = -2$ or 4 and $y = 3$ or 5

$$m = \frac{y}{x} = \frac{(3-5)}{(-2-4)} = \frac{-2}{-6} = \frac{1}{3}$$
 ✓

where $y = mx + c$

so $3 = \frac{1}{3}x + 2 + c$ ✓

$3 = -2/3 + c$ ✓

$-2 = c$ ✗

$\therefore y = \frac{1}{3}x - 2$ ✗

3 ✓

b) where $m = -3$ $x = 2$ $y = 3$

$3 = -3 \times 2 + c$ ✓

$3 = -6 + c$ ✓

$9 = c$ ✓

$\therefore y = -3x + 9$ ✓

5 ✓

8. Where $a = 1.50$ ~~b~~ $n = 50$ $d = 0.70$

$$a + (n-1) \times d = 1.50 + (50-1) \times 0.70 = \underline{\underline{\pounds 35.80}}$$

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9. where $a = 10$ $n = 5$

$$\text{so } ar^{(n-1)} = 700$$

$$10r^{(5-1)} = 700$$

$$r^{(5-1)} = 70$$

$$r^4 = 70$$

$$r = \sqrt[4]{70}$$

$$1^{\text{st}} \text{ Speed} = 10 \sqrt[4]{70}^{(1-1)} = 10 \text{ RPM}$$

$$2^{\text{nd}} \text{ Speed} = 10 \sqrt[4]{70}^{(2-1)} = 29 \text{ RPM}$$

$$3^{\text{rd}} \text{ Speed} = 10 \sqrt[4]{70}^{(3-1)} = 84 \text{ RPM}$$

$$4^{\text{th}} \text{ Speed} = 10 \sqrt[4]{70}^{(4-1)} = 242 \text{ RPM}$$

$$5^{\text{th}} \text{ Speed} = 10 \sqrt[4]{70}^{(5-1)} = 700 \text{ RPM}$$

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